Numerical simulation of bistatic scattering from a target at low altitude above rough sea surface under an EM wave incidence at low grazing angle by using the finite element method

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Abstract To study bistatic scattering from a target at low altitude above two-dimensional (2D) randomly rough sea surface under an electromagnetic (EM) wave incidence at low grazing angle (LGA), a numerical approach of the finite element method (FEM) is developed. The conformal perfectly matched layer (PML), as the truncation boundary of the FEM, is employed to reduce the reflection error of planar PML in conventional FEM. Numerical code of our FEM is examined by available solution of the forward backward iterative (FBM) method. Bistatic and back-scattering from composite model of a target above random rough sea surface generated by Monte Carlo realization, and functional dependence upon the sea surface wind speed, target altitude, incident and scattering angles, etc. are numerically simulated and discussed. This paper presents a numerical description of the observation principle and physical insight associated with the coupling interactions of a complex volumetric target and random rough sea surface.

Keywords: rough surface, low altitude target, EM scattering, LGA, FEM, PML.

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A study of scattering from a volumetric target above random rough surface is of great significance in target detection in terrain and oceanic clutters. Most of analytical approaches to rough surface scattering are based on some approximations, such as the Kirchhoff tangent plane approximation, small perturbation approximation, two-scale model and others. Analytical study of scattering from an object above the surface has been limited to the model of a sphere or a cylinder above a planar conducting surface. Numerical model for simulation of scattering from complex target and rough surface should have the aid of computational methods of electromagnetics, e.g. integral equation method, finite element method, finite difference time domain method (FDTD), as well as widely discussed forward backward method (FBM).
Because the integral equation is numerically discretized to produce a dense matrix to be solved, we need accurate and fast algorithm to accelerate computation and improve the accuracy and efficiency, especially for EM incidence at LGA. When a target is above rough surface, the scattering behavior involves the coupling interactions of a complex volumetric object and rough surface. The FEM can accommodate these multiple interactions in a tractable way. The approaches to FEM and FDTD have been limited to the scattering problem of a single rough surface\cite{4,5,6}, or an object buried beneath a rough interface at EM normal incidence\cite{10,11}. Numerical simulation of bistatic scattering from composite model of a target above rough surface remains a pending problem.

In this paper, a composite model is numerically developed by using the FEM method to study scattering from a target at low altitude above rough sea surface under EM wave incidence at LGA. The conformal PML\cite{12,13} is employed as the truncation boundary of FEM to reduce the reflection errors\cite{9} caused by conventional planar PML at LGA. Numerical code of our FEM is examined by available solution of the FBM. Bistatic and back-scattering from the composite model of a target above random rough sea surface, and functional dependence upon the sea surface wind speed, target altitude, incident and scattering angles, etc. are numerically simulated and discussed. This study presents a numerical description of the observation principle and physical insight associated with the coupling interactions of a complex target and random rough sea surface.

1 A composite model for a target at low altitude above rough surface

Conventional FEM usually employs an open-region boundary condition to truncate the size of computation region. The PML in our approach acts as a layer of lossy anisotropic medium surrounding the target/surface region, and absorbs the outgoing wave with no-reflection as in free space. It makes the FEM computation within a finite region.

As shown in fig. 1, there is a target located at \((x = x_0, z = h)\) above a 2D rough surface with random height \(z = f(x)\) and the mean value \(\langle f(x) \rangle = 0\). The rough surface is generated by using the Monte Carlo method in the Pierson-Moskowitz (P-M) sea spectrum\cite{15}. The conformal or the planar PML encloses the target/surface region. The surface length \(L\) is illuminated by a tapered EM wave incidence; \(\Gamma_p\) and \(\Gamma_c\) are the far-field integration paths for planar and conformal PML, respectively. Here \(h'\) is the path height in horizontal direction.

Scattering wave above the rough surface, including the PML region, satisfies the vector wave equation

\[
\nabla \times \left( \frac{1}{\mu_r} \bar{\Lambda}^{-1} \cdot \nabla \times \bar{E}_s \right) - k_0^2 \varepsilon_r \bar{\Lambda} \cdot \bar{E}_s = 0,
\]

where \(\bar{E}_s\) is the scattering electric field; \(\varepsilon_r, \mu_r\) are the relative permittivity and permeability, respectively; \(k_0\) is the wave number in free space. The tensor \(\bar{\Lambda}\) is the constitut-
On the perfectly electric conducting (PEC) surfaces of the target/rough surface, the boundary condition yields

\[ \hat{n} \times \vec{E}_s = -\hat{n} \times \vec{E}_i, \tag{2} \]

where \( \vec{E}_i \) is the incident electric field, and \( \hat{n} \) is normal vector of the surface. At the bottom interface of PML, as shown in fig. 1, the scattering fields are completely absorbed

\[ \hat{n} \times \vec{E}_s = 0 \tag{3} \]

Eqs. (1)—(3) are to be solved for our scattering model of fig. 1.

According to the generalized variational principle\[14\], solving the boundary-value problem of eqs. (1)—(3) is equivalent to seeking the stationary point of the functional given by

\[ F(\vec{E}_s) = \frac{1}{2} \iiint \left[ \frac{1}{\mu} (\nabla \times \vec{E}_s) \cdot \overline{\Lambda}^{-1} (\nabla \times \vec{E}_s) - k_0^2 \varepsilon_0 \vec{E}_s \cdot \overline{\Lambda} \cdot \vec{E}_s \right] dv. \tag{4} \]

In the local orthogonal coordinates \((\xi_1, \xi_2, \xi_3)\), \( \overline{\Lambda} \) is expressed in the form of uni-axial tensor

\[ \overline{\Lambda}_{\xi_1, \xi_2, \xi_3} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \tag{5} \]

It can be proved that (5) is a general expression for both planar and conformal PML. By using the Jacobi matrix \( \overline{J} \), the tensor \( \overline{\Lambda} \) is transformed from the local coordinate to global Cartesian coordinate as follows:
In our 2D model of rough surface under a horizontally (TE) polarized wave incidence, we have

$$\nabla \times \mathbf{E}_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}_{x,z} \nabla E_{sy}. \quad (7)$$

Assuming the local $\xi_3$ axis as the $\hat{y}$-direction of the principal coordinate, $\vec{\Lambda}$ can be rewritten as

$$\vec{\Lambda}_{x,z,y} = \begin{bmatrix} \tilde{\Lambda} & 0 \\ 0 & \beta \end{bmatrix}_{x,z,y}, \quad (8)$$

where $\tilde{\Lambda}$ is a $2 \times 2$ sub-matrix and $\hat{J}$ is a 2D Jacobi matrix. Then eq. (4) becomes

$$F(E_{sy}) = \frac{1}{2} \int_{\Omega} \left( \frac{1}{\mu_r} \nabla E_{sy} \cdot \tilde{\Lambda} \cdot \nabla E_{sy} - k_0^2 \varepsilon_r \beta E_{sy}^2 \right) d\Omega. \quad (10)$$

By eqs. (2) and (3), PEC surface and PML bottom surface satisfy the boundary conditions

$$E_{sy} \bigg|_{\text{PEC}} = -E_{y0} \bigg|_{\text{PEC}}, \quad (11)$$

$$E_{sy} \bigg|_{\text{PML}} = 0. \quad (12)$$

The incident field $E_{y0}$ is taken as a Thorsos tapered wave

$$E_{y0} = e^{-x+2zg\theta}/g^2 \ e^{-j k_0 (\sin \theta - \cos \theta)/(1+w)}, \quad (13)$$

where $g$ is the tapering parameter and

$$w = [2(x+zg\theta)^2/(g^2-1)](k_0 g \cos \theta)^2. \quad (14)$$

Evaluating the stationary point of the functional problem of eqs. (10)–(12) by using the FEM, the scattering field at each point of the computation region can be solved. The bistatic and back-scattering coefficients are obtained.

The bistatic scattering coefficient, i.e. bistatic RCS (radar cross section) for tapered wave incidence is defined as

$$F(E_{sy}) = \frac{1}{2} \int_{\Omega} \left( \frac{1}{\mu_r} \nabla E_{sy} \cdot \tilde{\Lambda} \cdot \nabla E_{sy} - k_0^2 \varepsilon_r \beta E_{sy}^2 \right) d\Omega. \quad (10)$$
\[ \sigma = \lim_{r \to \infty} 2\pi r \frac{|E_{yy}|^2}{P_i}, \]  

where

\[ P_i = \sqrt{\frac{\pi}{2}} g \cos \theta_i \left( 1 - \frac{1 + 2t_g^2 \theta_i}{2(k_0 g \cos \theta_i)^2} \right). \]  

2 Conformal PML absorbing boundary

The perfectly matched layer (PML) is recently proposed by Berenger\[9\]. An ideal PML might be non-reflectional for any incident angle and broad frequency spectrum, and can absorb all outgoing waves. Usually, the PML takes the computation region as a rectangular box. It might cause the reflection error due to the abrupt change in geometrical and physical parameters around the corners intersected by several pieces of planar PML. Meanwhile, to contain the target one has to increase the height of horizontal integral path \( h' \). As the wave is incident at LGA, the main lobe of scattering pattern from rough surface intersects the PML corners and causes large system error.

The conformal PML\[12\] is employed to circumvent this problem. As shown in fig. 1, a quarter of elliptical shaped conformal PML at the end of horizontal planar PML is patched to the corner regions.

The conformal PML was presented by Teixeira\[12\] in terms of anisotropic tensor. The planar, cylindrical, and spherical PML are actually the special cases of the conformal PML. As shown in fig. 2, the conformal PML is a smooth convex shell of anisotropic medium. Here the so-called conformal means that the inner interface \( S \) of the conformal PML is parallel to the target surface or a fictitious surface \( \Gamma_c \) surrounding the target. At each point of the parallel surface, the constitutive parameters of the anisotropic conformal PML is given by \( \mu = \mu, \lambda \) and \( \varepsilon = \varepsilon, \lambda \), where
\[
\overline{\lambda} = \xi_1 \xi_1 \left( \frac{s_2 s_3}{s_1} \right) + \xi_2 \xi_2 \left( \frac{s_1 s_3}{s_2} \right) + \xi_3 \xi_3 \left( \frac{s_1 s_2}{s_3} \right). \]

In the cross-section of x-z plane, the \( \xi_1 \)- and \( \xi_2 \)-axes are tangential and perpendicular to the surface \( S \), respectively. The parameter \( t \) is the thickness of conformal PML, \( d \) is the distance between \( S \) and \( \Gamma_c \), and \( \xi_2 = 0 \) on \( S \). The definition and realization of 3D conformal PML have been discussed in refs. [12,13]. Fig. 2 shows a special case\[13\]
where the curvature radius in \( \hat{y} \)-direction is infinite. In our model, we obtain

\[
\begin{align*}
    s_1 &= 1 - j\frac{\xi_1^2}{(3t^2r_{\xi_1})}, \\
    s_2 &= 1 - j\frac{\xi_2^2}{(t^2)}, \\
    s_3 &= 1,
\end{align*}
\]

where the constant \( \gamma \) determines the attenuation rate of the PML; \( r_{\xi_1} \) is the curvature radius in the \( \xi_1 \)-direction at the point \( P(\xi_1, \xi_2, \xi_3) \) on the parallel surface. In order to make smooth change between the interface of the PML and free space, the imaginary part of \( s_2 \) is chosen as a second order taper function. Let \( r_{\xi_1} = \infty \). Then eq. (17) is reduced to the form of planar PML.

Absorption of conformal PML can be improved by increasing \( t \) and \( \gamma \). But, the increase of these two parameters can slow down the FEM speed, because increasing \( t \) yields larger \( \Omega \) of eq.(10) and larger scale of FEM equation. On the other hand, the condition number of the FEM system matrix is increased with larger \( \gamma \) and the iterative convergence becomes much slower. Thus small values of \( t \) and \( \gamma \) to ensure enough absorption and can be empirically determined for the purpose of modeling.

3 Numerical results and discussions

We are concerned with a tapered plane wave incident upon a 2D perfectly conducting rough surface generated by one Monte Carlo realization in the P-M spectrum. The tapering parameter \( g = L/6 \) and the wavelength \( \lambda = 1 \text{ m} \). The parameters of the PML are chosen as \( d = 0.1\lambda \), \( t = 0.2\lambda \), and \( \gamma = 18 \). The axial ratio of elliptical conformal PML is 6 while the major axis of ellipse coincides with the horizontal plane.

Our FEM code is first examined by available solution of the FBM method. Fig. 3 gives the bistatic scattering coefficient of a conducting flat surface under EM incidence at LGA \( \theta_i = 80^\circ \). Other parameters are chosen as \( h' = 3\lambda \), \( L = 300\lambda \). It can be seen that the result of the FEM with the planar PML causes too high a scattering around backscattering angle \((-80^\circ)\), which is due to error on the edge region of the planar PML. The FEM with conformal PML not only eliminates the reflection error, but also significantly improves the accuracy for all scattering angles. Certainly, there exists only specular reflection from the flat surface. However, calculations of fig. 3 can be used as an accuracy test of different approaches.

Fig. 4 shows bistatic scattering from rough sea surface driven by strong wind speed \((u = 10 \text{ m/s})\), where \( L = 409\lambda \) and conformal PML is applied. Incident angle is also \( \theta_i = 80^\circ \). Thus our FEM results are well validated by the FBM.

We are now concerned with a model of an airfoil target above rough surface. Sup-
Numerical simulation of bistatic scattering from a target at low altitude above rough sea surface 299

Fig. 3. Bistatic scattering from conducting flat surface by using three methods.

Fig. 4. Bistatic scattering from rough sea surface using the FEM and FBM methods.

pose that this NACA0012 four-digital airfoil has a chord length 2m, a leading edge at $x_0 = 0$ m and the center axis at the height $h = 1$ m. The rough surface $L = 409.6\lambda$ and incident $\theta_i = 80^\circ$. Fig. 5 shows bistatic scattering when: (i) the target is in the free space, (ii) the target is above the conducting flat surface, and (iii) the target is above rough sea surface driven by sea surface wind speed $u = 3$ m/s.

It can be seen that angular scattering from the target above a rough surface at such a low wind speed is quite similar to the case of the target above a flat surface. Analysis of the image method is applicable. These results are obtained by using one Monte Carlo

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realization. If the result is averaged over multi-realizations, the fluctuation of scattering pattern should be more further smoothed.

Figs. 6(a)–(c) shows bistatic scattering from a target at different altitude \( h \) above the rough sea surface: (a) conducting flat surface, (b) rough sea surface with \( u = 3 \) m/s, (c) rough sea surface with \( u = 10 \) m/s. The parameters of target and incident wave are the same as in fig. 5.

In fig. 6 (a) and (b), the fluctuation of bistatic scattering versus scattering angle is largely regular, and the target altitude affects the angular scattering pattern. This phenomenon is caused by interaction between the target and its mirror image. However, as the sea surface becomes rougher at strong surface wind speed (fig. 6(c)), angular pattern of bistatic scattering becomes more complicated, i.e. not regularly oscillated.

Based on the image method and two-source array radiation principle, the phase difference \( \Delta \Phi \) between single-scattering fields of the target and its mirror image is written as

\[
\Delta \Phi = 2k_0 h \cos \theta_s .
\]

Angular variation of \( \Delta \Phi \) with respect to \( \theta_s \) is

\[
d(\Delta \Phi) / d\theta_s = 2k_0 h \sin \theta_s .
\]

It means that \( d(\Delta \Phi) / d\theta_s \) becomes larger with increasing \( h \) and \( \theta_s \).

Theoretically, when the target altitude is varying, scattering would be periodically fluctuating. Fig. 7 shows backscattering coefficient versus target altitude above different
Numerical simulation of bistatic scattering from a target at low altitude above rough sea surface

Fig. 6. Bistatic scattering of the target at different altitudes above the surface. (a) Conducting flat surface; (b) rough sea surface with $u = 3$ m/s; (c) rough sea surface with $u = 10$ m/s.
rough sea surfaces. The periodic fluctuation in backscattering can be identified, especially for a flat surface \( u = 0 \) m/s or sea surface of \( u = 3 \) m/s. From eq.(21), it is derived that the phase cancellation meets

\[
h = \frac{n\pi}{2k_0 \cos \theta}, \quad n = 1, 2, 3, \ldots
\]

(23)

where the odd and even \( n \) correspond to the maximum and minimum of backscattering, respectively. As the target altitude becomes higher, the target moves from the center of the tapered incident wave and might reduce the scattering maximum.

At strong surface wind speed, say, \( u = 10 \) m/s, the wave height of the sea surface might reach as high as 3.0 m. In fig. 7, the target altitude above the surface with \( u = 10 \) m/s starts from 1.7 m. Fig. 7 shows backscattering fluctuation when the target altitude changes.

![Fig. 7. Backscattering coefficient versus target altitude (\( \theta_i = 80^\circ \)).](image)

Backscattering coefficient from the sea surface with low wind speed without target is around \(-40\) dB at LGA incidence (fig. 5). In other words, if the target echo is lower than \(-40\) dB, the target would not be detectable. The area enclosed by the dotted lines in fig. 7 might be regarded as blind altitude (B). This altitude is certainly related to the incident angle \( \theta_i \) and surface roughness.

As the incident angle changes, backscattering from the target, and target/rough sea surface of \( u = 3 \) m/s and 10 m/s is presented in fig. 8. The target is located at \( x_0 = 0 \) m and \( h = 3 \) m, and other parameters are the same as in fig. 7.

It can be seen that LGA incidence causes more fluctuation of backscattering, especially for rough surface with low surface wind speed.

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The size of airfoil target in this paper is electrically small (length $2\lambda$ and thickness $0.24\lambda$). Scattering includes single scattering from the target, and surface and multi-interactions between them. Angular scattering pattern of an electrically small target is dominated by forward scattering. When the EM wave is incident at a small angle, interaction of the target and surface is contributed mainly by the forward scattering of the target. It is quite different from the case of large angle incidence where the side-lobe of scattering pattern of the target causes interaction with the surface. It is one of the reasons why the maximum level of scattering at small angle incidence is always higher than large angle incidence. It also explains why the fluctuation amplitude at small incidence angle is smaller.

However, if the target size becomes larger, the angular scattering pattern becomes more complicated and no simple conclusion like this can be drawn. Following the analytic study and numerical code of ref. [3] we calculated the scattering from an electrically large, perfectly conducting cylinder above conducting flat surface. The results support our viewpoint. A study on scattering from a large target above rough surface by using the parallel algorithm and domain decomposition method is in progress.

4 Conclusions

An FEM approach is developed to study bistatic scattering from a target at low altitude over 2D randomly rough sea surface at LGA incidence. Rough sea surface is generated by the Monte Carlo realization in P-M sea spectrum. By using the conformal PML absorbing boundary in the FEM, the reflection error caused by the PML intersection region is eliminated, and accurate calculation for the composite modeling can be achieved. Bistatic and backscattering from the composite model of an airfoil target above randomly rough sea surface are numerically simulated. The functional depend-
ence of angular scattering upon the sea surface wind speed, target altitude, incident and scattering angles, etc. are numerically demonstrated and discussed. This study presents a numerical description of the observation principle and a physical insight into scattering and coupling interaction from a complex volumetric target above random rough sea surface.

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